Computation of Shock-Sound Interaction Using Finite Volume Essentially Nonoscillatory Scheme

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Introduction

IN an imperfectly expanded supersonic jet, shock-cell structures are formed downstream of the nozzle exit. The unstable motion of the shock-cell caused by the shear layer generates acoustic waves, that is, jet noise. Sometimes, the acoustic waves interact with the shock. To simulate the jet noise correctly, the scheme for the computation of the shock-sound interaction should be validated accurately. However, an analysis of the shock-sound interaction problem for which an analytic solution exists has not been attempted until recently.[1]

Therefore, the subject of this Note is to validate the accuracy of the present finite volume essentially nonoscillatory (ENO) shock-capturing scheme through the computation of the shock-sound interaction problem. The major technical difficulty in this problem is to capture accurately the discontinuity in the steady-mean flow such as a shock and a perturbed wave from the shock-sound interaction.

A class of uniformly high-order-accurate, finite volume ENO scheme has been developed by Harten et al.[2] An attempt to apply the ENO scheme to aeroacoustic problems was made by Meadows et al.,[3] who discussed spurious entropy waves in calculations of an unsteady shock in the flowfield. This conventional ENO scheme has two problems. One of the problems is that the computational time for the simulation of complex geometries is quite long and the other is the convergence problem that affects long-time steady calculations. For reducing the computational time, the modified flux approach (MFA) scheme that is introduced by Yang and Hsu[4] is adopted for the present study. The authors have experience with a complicated system like a reciprocating engine intake problem with a moving piston and a valve that was successfully simulated using the present MFA-type finite volume ENO scheme.[5]

The conventional ENO scheme for high-order accuracy and nonoscillatory shock capturing is achieved through the use of adaptive stenciling. One of the methods of improving the convergence of the ENO scheme is to employ the biased-stencil algorithm. Atkins[6] has proposed a linear biased-stencil algorithm in order to obtain a linear perturbed solution without a mean flow, a steady solution without a shock, and a perturbed solution from initially steady flow in a converging-diverging nozzle. Casper and Meadows[7] have also suggested a nonlinear biased algorithm that retains the nonbiased stencils in smooth regions, yet allows more freedom near a discontinuity. In this case only a steady flow with a shock in a nozzle was calculated, although a perturbed solution as a result of a shock-sound interaction was not obtained.

The purpose of the present Note is to obtain a suitably converged steady solution with a shock in a nozzle and a perturbed solution from a shock initially existing in the mean flow, using both a linear and a nonlinear biased-stencil algorithm. It is shown that the present calculation results are in excellent agreement with analytic ones.

Numerical Method

The finite volume ENO scheme used for the present work is briefly described as follows. The conservative forms of quasi-one-dimensional Euler equations in generalized coordinates are represented as

$$\frac{\partial}{\partial t}\left(\frac{AQ}{J}\right)+\frac{\partial}{\partial \xi}(AE)=\frac{H}{J} \tag{1}$$

where

$$Q=\begin{bmatrix}\rho\\ \rho u\\ \rho e_t\end{bmatrix},\qquad E=\begin{bmatrix}\rho u\\ \rho u^2+p\\ (\rho e_t+p)u\end{bmatrix},\qquad H=\begin{bmatrix}0\\ p\dfrac{\mathrm{d}A}{\mathrm{d}x}\\ 0\end{bmatrix} \tag{2}$$

The variables ρ, u, p, e_t, and A are the density, velocity, pressure, total energy, and nozzle area, respectively; and p is related to other variables by $p=(\gamma-1)[\rho e_t-\rho u^2/2]$, where γ is the ratio of specific heats. The Jacobian of transformation J is ξ_x in the one-dimensional problem. The spatial derivative term $\partial(AE)/\partial\xi$ in Eq. (1) can be expressed as based on Roe's approximate method:

$$\frac{\partial(AE)}{\partial \xi}=\left[(A\tilde{E})_{j+\frac{1}{2}}-(A\tilde{E})_{j-\frac{1}{2}}\right] \tag{3}$$

where $\tilde{E}_{j+1/2}$ is the numerical flux defined by

$$\tilde{E}_{j+\frac{1}{2}}=\tfrac{1}{2}\left(E_j+E_{j+1}+R_{j+\frac{1}{2}}\cdot\Phi_{j+\frac{1}{2}}/J_{j+\frac{1}{2}}\right) \tag{4}$$

and $j+\frac{1}{2}$ denotes the cell interface value and is obtained by Roe's average. The R is the right eigenvector matrix, and the components of the column vector $\Phi_{j+1/2}$ in Eq. (4) are shown in Ref. 5.

The adaptive stenciling that adapts its interpolation set to the smoothest available part of the solution makes ENO schemes highly nonlinear. However, these schemes also have certain drawbacks. The free adaptation of stencils is not necessary in regions where the solution is smooth. These drawbacks can be remedied through the use of a biased-stencil algorithm. In this Note Atkins's linear biased-stencil algorithm and Casper's nonlinear one were tested for the shock-sound interaction problem, and these algorithms have been slightly modified for applying to the present finite volume ENO scheme. Both of them show satisfactory results of the steady solution with a shock and a perturbed solution from initially steady flow compared with the analytic ones. However, the linear algorithm has better convergent history than the nonlinear case, as shown in Fig. 1.

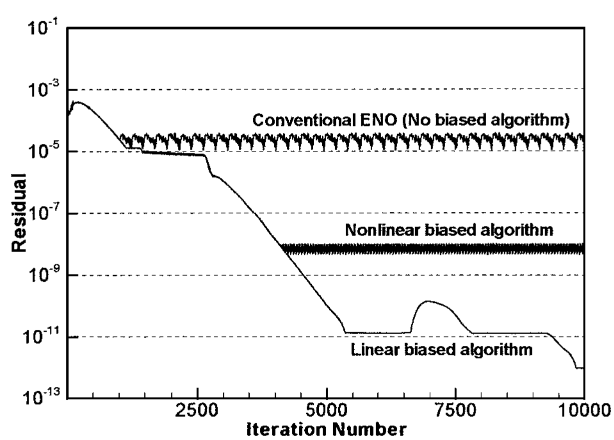

Fig. 1 Residual of the steady-state solution.

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Numerical Results and Discussion

For quasi-one-dimensional transonic nozzle computations, problem 2 category 1 in the Third CAA Workshop on Benchmark Problems[1] is solved. This problem is to simulate the shock-sound interactions in a transonic nozzle. The domain is $-10 < x < 10$, and the area of the nozzle is given by

$$A(x) = \begin{cases} 0.536572 - 0.198086\exp\left[-(\ln 2)\left(\dfrac{x}{0.6}\right)^2\right], & x > 0 \\ 1.0 - 0.661514\exp\left[-(\ln 2)\left(\dfrac{x}{0.6}\right)^2\right], & x < 0 \end{cases} \tag{5}$$

At the inlet boundary the conditions are

$$\begin{bmatrix} \rho \\ u \\ p \end{bmatrix} = \begin{bmatrix} 1 \\ M \\ 1/\gamma \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \varepsilon \cdot \sin\left[\omega\left(\frac{x}{1+M} - t\right)\right] \tag{6}$$

where $\varepsilon = 1.0 \times 10^{-5}$, $\omega = 0.6\,\pi$, and $M_{\text{inlet}} = 0.2006533$. The pressure will be set at the outflow boundary to create a shock, $(p)_{\text{exit}} = 0.6071752$.

A steady-state solution as shown in Fig. 2 is obtained with a third-order ENO scheme with biased-stencil algorithm. This numerically converged initial condition could not be obtained with a conventional ENO scheme. A suitably converged solution demonstrates that the linear and the nonlinear biased-stencil algorithms are well applied to the present finite volume ENO scheme.

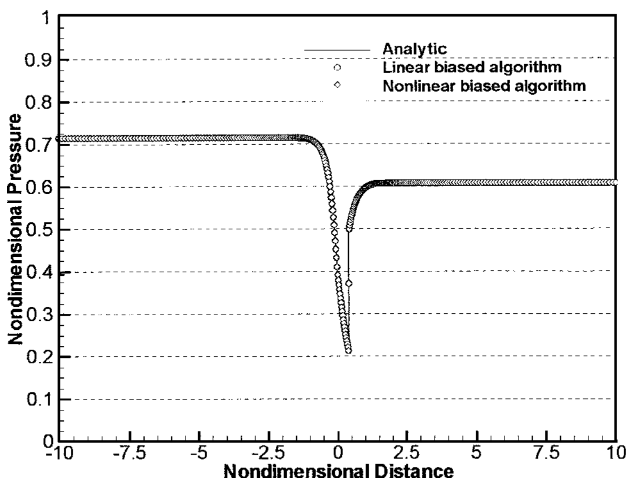

Fig. 2 Initial steady-state solution.

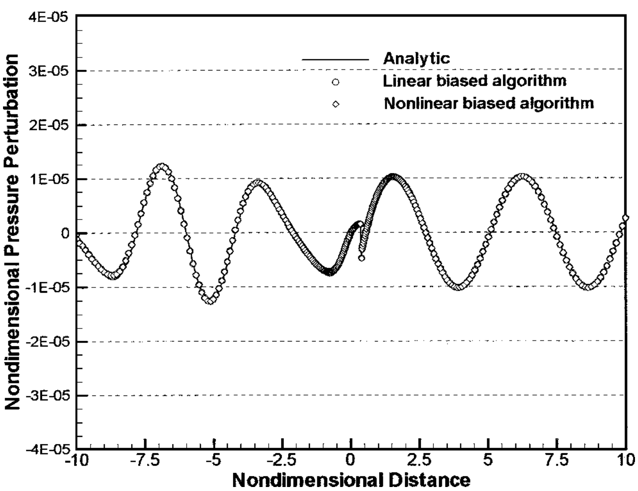

Fig. 3 Spatial perturbed pressure distribution.

After the steady state is achieved, an acoustic disturbance is introduced at the inlet, $x = -10$. The calculation is performed on a 301 cell clustered near the nozzle throat. The time step used is determined by the Courant–Friedrichs–Lewy condition with a Courant number of 0.9. The inflow is perturbed for $0 < t/T_\lambda \leq 50$, where $T_\lambda = 2\pi/\omega$ is one period of the incoming acoustic wave. Nonreflecting transparent characteristic boundary condition is used as the numerical boundary conditions at the inflow and outflow for both the steady-state solution and the time-dependent solution.[8] This condition enables the source to be transparent and maintains the nonreflection at the inlet boundary. Figure 3 shows the spatial perturbed pressure distribution at the start of a period $[x, p(x) - \bar{p}(x)]$ over the period of the perturbation. The acoustic wave propagates to the shock wave, and a reflected wave and a transmitted wave are formed. It is observed that a large amplitude is generated at the shock position interacting between acoustic wave and shock wave. The results in Figs. 2 and 3 are in good agreement with the analytical solutions that were provided by the committee of Third CAA Workshop on Benchmark Problems.[1]

Conclusions

The modified-flux-approach-type finite volume ENO scheme with the linear and the nonlinear biased-stencil algorithms is successfully applied to the shock-sound interaction problem, which had not been solved previously using the ENO scheme. On the basis of the simulations, we obtained a suitably converged solution with a shock. Other researchers have already computed it. However, we computed accurately a perturbed solution from a shock initially existing in the mean flow using the upwind ENO scheme for the first time. The solutions are in excellent agreement with analytical ones, and it is shown that the linear algorithm has a better convergence history than the nonlinear case.

We have found that the nonreflecting transparent characteristic boundary condition is necessary when inflow perturbations are imposed on the inflow boundary. Finally, it is concluded that the proposed finite volume ENO scheme with the biased-stencil algorithm could assist in investigating practical aeroacoustic problems that involve shocks.

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